Lab Activity 3

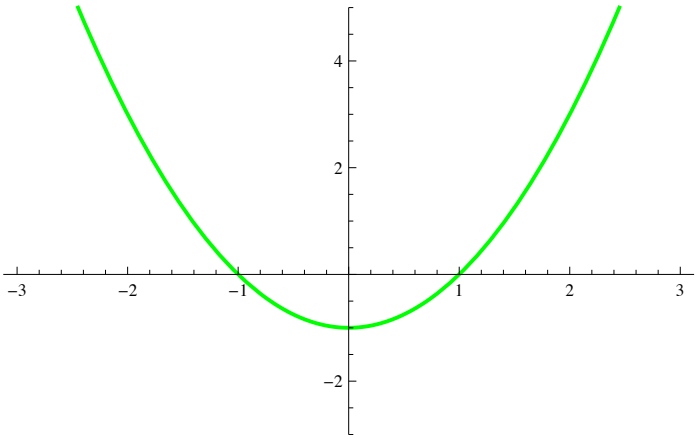
Piecewise Functions

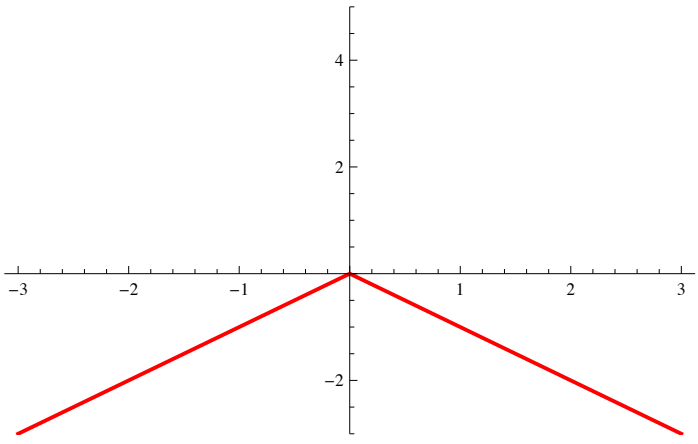
Graphing Piecewise functions

Suppose we wish to graph the function

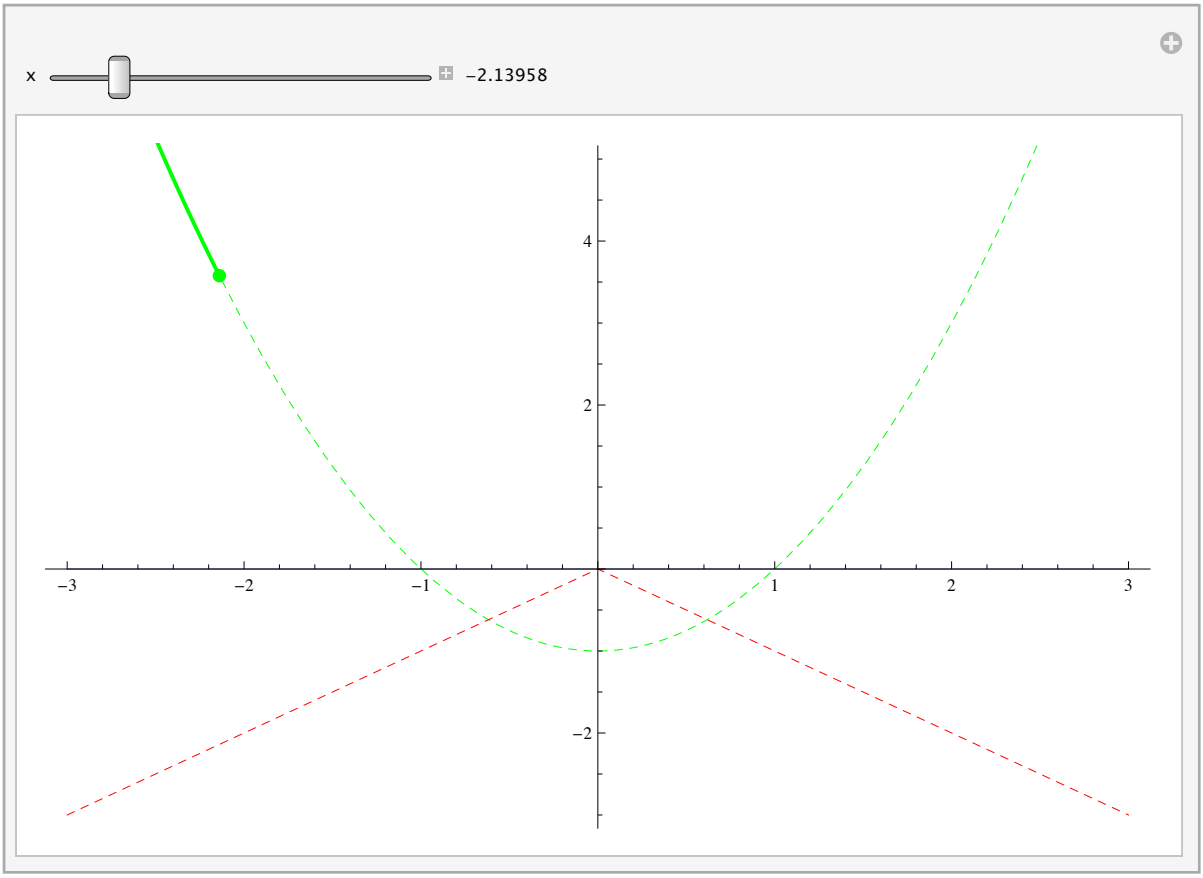
$$f(x) = \begin{cases} x^2 \text{ if } x \leqslant -1 \\ -|x| \text{ if } x > -1 \end{cases}$$

The graph of the two *component functions* are below: $y = x^2$ is in green and $y = -|x|$ is in red.

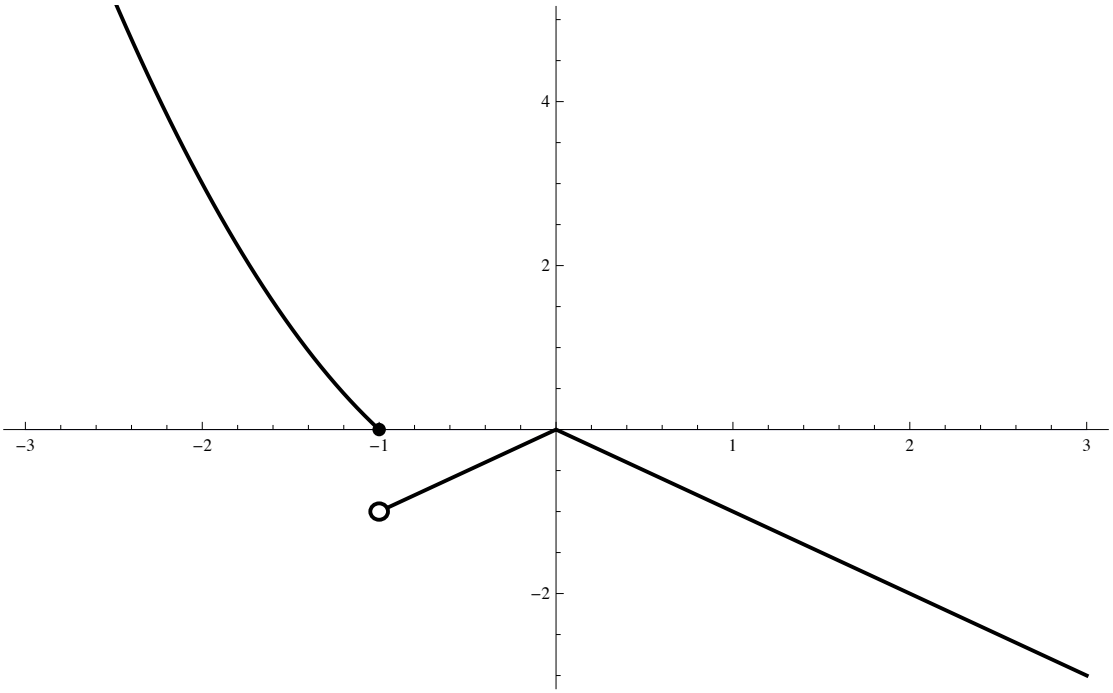




Below, both our component functions are graphed in dashed strokes on the same set of axes. Move the slider to the right. As x increases (the black dot moves to to right), the graph of the piecewise function will be drawn in solid strokes. Note how, at $x = -1$, the graph jumps from $y = x^2$ to $y = -|x|$.



Removing the dashed curves from the final result (since they are not part of the graph), we obtain the following graph for $f(x) = \begin{cases} x^2 \text{ if } x \leqslant -1 \\ -|x| \text{ if } x > -1 \end{cases}$:

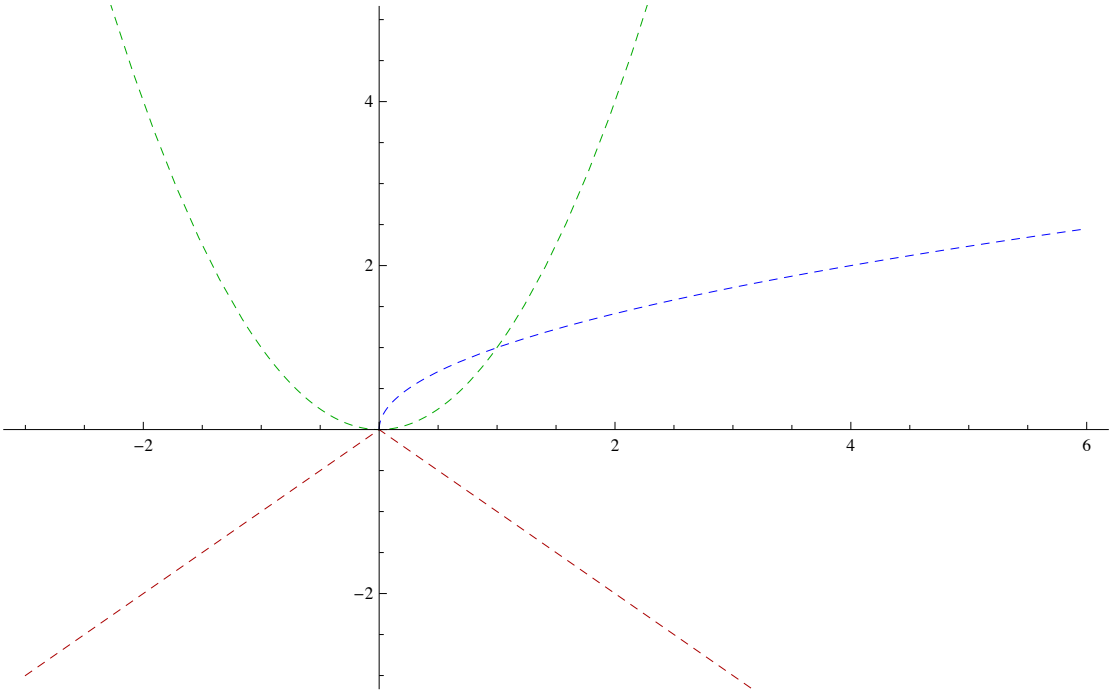


How do we know where to draw solid circles and open circles? These circles mark where one component function's graph ends and the other begins. We switch from one component function to the other at $x = -1$. Since our piecewise function is still equal to x^2 when $x = -1$, and the point $(-1, 0)$ is on the graph of $y = x^2$, we draw a **closed** circle at $(-1, 0)$. We draw an **open** circle at $(-1, -1)$ since the point $(-1, -1)$ is on the graph of $y = -|x|$, and since our piecewise function is equal to $-|x|$ when $x > -1$ but not when $x = -1$.

Problem 1.

Sketch the graph of the piecewise function $g(x) = \begin{cases} x^2 \text{ if } x \leq -1 \\ -|x| \text{ if } -1 < x < 4 \\ \sqrt{x} \text{ if } x \geq 4 \end{cases}$

The graphs of all three component functions are in dashed strokes on the same set of axes below. Draw over the dashed graphs appropriately, and add closed/open circles where needed.

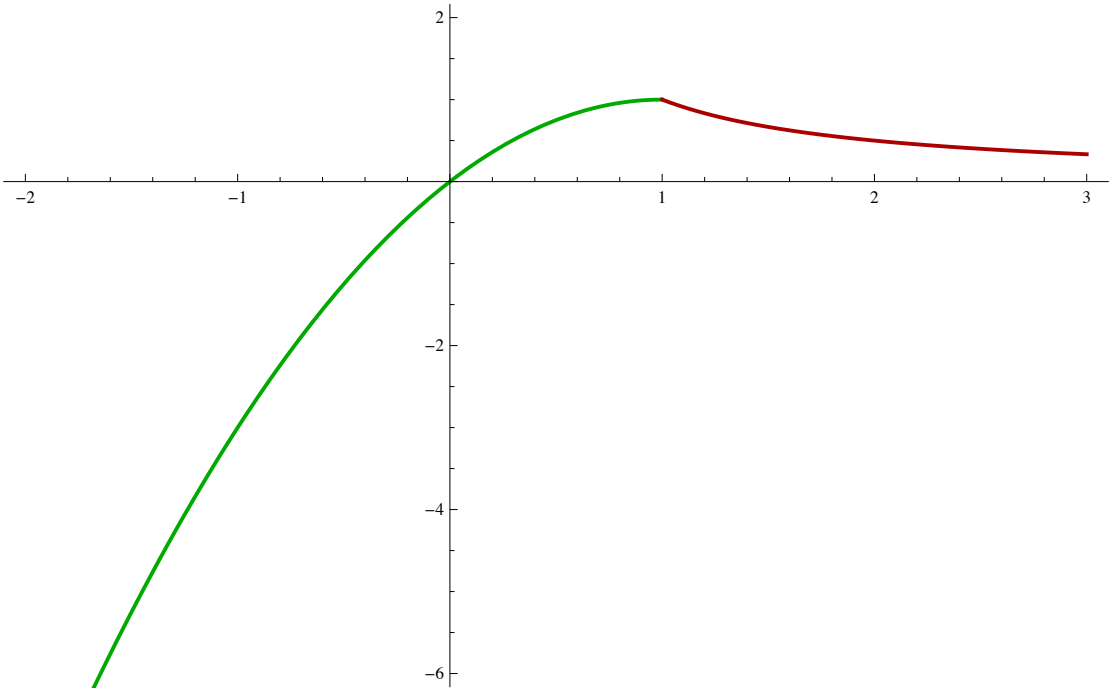


Evaluating Piecewise Functions

Consider the piecewise function

$$f(x) = \begin{cases} -(x-1)^2 + 1 \text{ if } x \leq 1 \\ \frac{1}{x} \text{ if } x > 1 \end{cases}$$

Skill Check. Sketch the graph of the piecewise function. It should match the graph below. (Note: when $x = 1$, $-(x-1)^2+1 = \frac{1}{x}$, so the graphs of the component functions meet up at $(1, 1)$.)

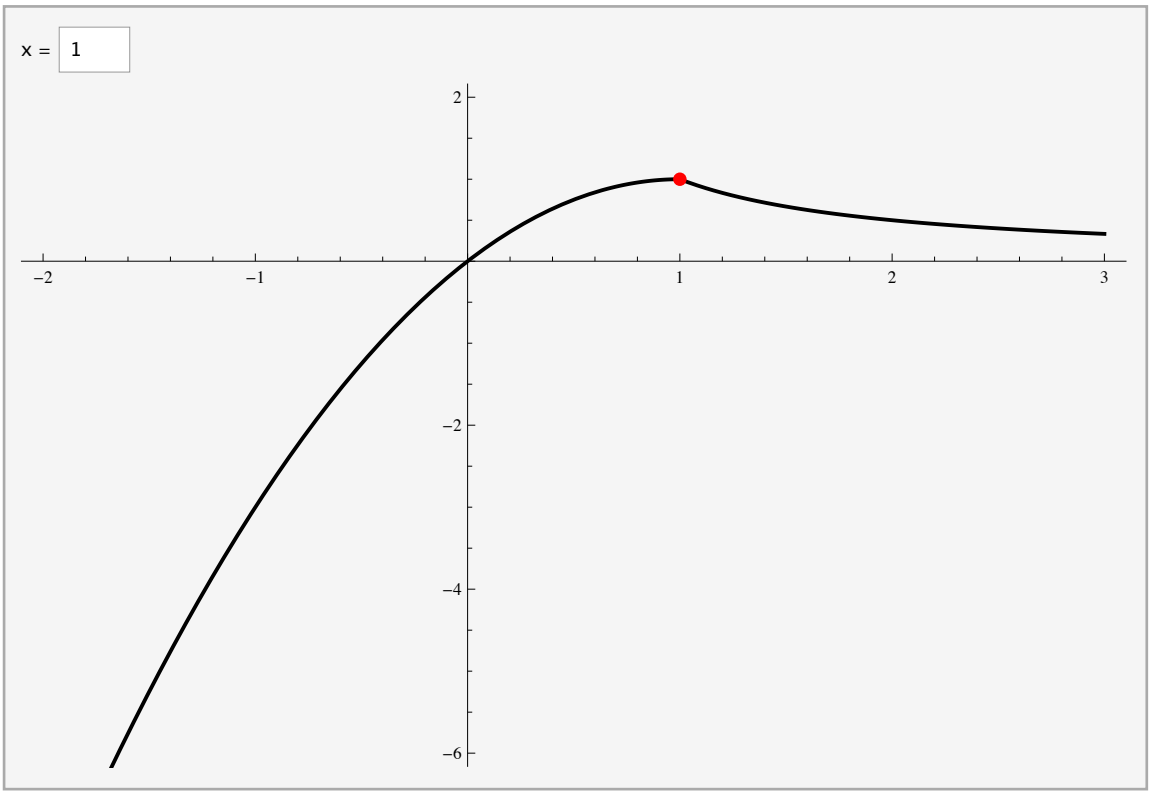


Problem 2.

Now we will evaluate the piecewise function $f(x) = \begin{cases} -(x-1)^2 + 1 \text{ if } x \leq 1 \\ \frac{1}{x} \text{ if } x > 1 \end{cases}$ for various values of x.

For each value of x in Table 1:

- Type the value in the input box for the graph below and press the enter key (the left-hand enter key, not the one on the keypad). A red point will appear on the graph at that x-value.
- Using the graph, estimate the value of $f(x)$. Write your answer in Table 1.
- Select the component function whose graph appears at that x-value. (either $-(x-1)^2 + 1$ or $\frac{1}{x}$)
- Plug the x-value into the appropriate component function and evaluate. Compare your answer to your estimated function value.



x − value	Estimated $f(x)$ value	Component Function	$f(x)$
0	0	$-(x-1)^2+1$	$-(0-1)^2+1=0$
-1			
0.5			
1.5			

Problem 3.

For $g(x) = \begin{cases} x^2 \text{ if } x \leq -1 \\ -|x| \text{ if } -1 < x < 4 \\ \sqrt{x} \text{ if } x \geq 4 \end{cases}$

Evaluate $g(-2)$, $g(-0.5)$, $g(4)$, and $g(5)$.